

Holographic superconductor models in the non-minimal derivative coupling theory

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Abstract

We study a general class of holographic superconductor models via the Stückelberg mechanism in the non-minimal derivative coupling theory in which the charged scalar field is kinetically coupling to Einstein's tensor. We explore the effects of the coupling parameter on the critical temperature, the order of phase transitions and the critical exponents near the second-order phase transition point. Moreover, we compute the electric conductive using the probe approximation and check the ratios ω_g/T_c for the different coupling parameters.

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I. INTRODUCTION

One of the most significant discovery in the string theory is the AdS/CFT correspondence [1–3] which relates a weakly coupled gravity theory in $d+1$ -dimensional AdS spacetime to a strongly coupled conformal field theory living on the d -dimensional boundary. It is widely believed now that the AdS/CFT correspondence could provide a new means of studying the strongly interacting condensed matter systems in which the perturbative methods are no longer valid. Thus, this holographic method is expected to give us some insights into the nature of the pairing mechanism in the high temperature superconductors which fall outside of the scope of current theories of superconductivity. In the pioneering papers [4–8], it was suggested that the spontaneous $U(1)$ symmetry breaking happened in the bulk black holes can be used to build gravitational duals of the transition from normal state to superconducting state in the boundary theory. This dual model consists of a system with a black hole and a charged scalar field, in which the black hole admits scalar hair at temperature T smaller than a critical temperature T_c . The appearance of a hairy AdS black hole means the formation of the scalar condensation in the boundary CFT, which indicates that the expectation value of charged operators undergoes the $U(1)$ symmetry breaking and then the phase transition is occurred. It was argued that this phase transition belongs to the second order. Due its potential applications to the condensed matter physics, holographic superconductors in the various theories of gravity have been investigated in the recent years [9–42]. Recently, Franco *et al.* [43, 44] investigated the general models of holographic superconductors in which the spontaneous breaking of a global $U(1)$ symmetry occurs via the Stückelberg mechanism. They found that both the order of the phase transition and the critical exponents for the second phase transition can be tuned by the parameters defined in the models. These generalized holographic superconductor models have also been extended recently to the Gauss-Bonnet gravity [45] and Born-Infeld electrodynamics [46].

It is well known that the properties of holographic superconductors depend on the coupling between the scalar field and the electromagnetic field. Theoretically, the general form of the action with more couplings can be expressed as

$$S = \int d^4x \sqrt{-g} \left[f(\psi, R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) + K(\psi, \partial_\mu\psi\partial^\mu\psi, \nabla^2\psi, R^{\mu\nu}\partial_\mu\psi\partial_\nu\psi, \dots) + V(\psi) + Y(\psi)\mathcal{L}_m \right], \quad (1)$$

where f , K and Y are arbitrary functions of the corresponding variables, \mathcal{L}_m is the Lagrangian for other matter fields. Obviously, the nonlinear functions f , K and Y provide the more non-minimal couplings among fields and the background spacetime. These new couplings modify the usual Klein-Gordon equation and

Einstein-Maxwell equation so that the motion equations for the scalar field and electromagnet field are no longer generally the second-order differential equations, which may changes the properties of holographic superconductors in the AdS spacetime. Recently, F. Aprile *et al.* [47] considered a special case of the action (1) in which the system contains the couplings among a scalar field, electromagnetic field and the cosmological constant so that the action has a form

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{4} G(\psi) F^{\mu\nu} F_{\mu\nu} + \frac{6}{L^2} U(\psi) - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{2} J(\psi) A_\mu A^\mu \right]. \quad (2)$$

They studied the properties of holographic superconductors and found that some aspects of the quantum critical behavior strongly depend on the choice of couplings $G(\psi)$, $U(\psi)$ and $J(\psi)$. Another interesting case is that the scalar field in the action (1) is kinetically coupling with Einstein's tensor. Sushkov found [48] that the equation of motion for the scalar field can be reduced to second-order differential equation in this model, which means that the theory is a “good” dynamical theory from the point of view of physics. The applications of this model to the cosmology has been done extensively in [48–51]. It is found that in cosmology the problem of graceful exit from inflation with such a coupling has a natural solution without any fine-tuned potential. The dynamical behaviors and Hawking radiation of a scalar field coupling to Einstein's tensor in the background of a black hole spacetime have been studied in [52, 52, 53]. It was showed that this coupling changes the stability of the black hole and enhances Hawking radiation of the black hole. The main purpose of this paper is to investigate the properties of holographic superconductors in the models in which a charged scalar field is kinetically coupling to Einstein's tensor and probe the effects of such a coupling on physical quantities and on the dynamics of the phase transition.

This paper is organized as follows. In Sec. II, we will study the scalar condensation and the phase transitions for a general class of the holographic superconductor models via the Stückelberg mechanism as the charged scalar field is kinetically coupling to Einstein's tensor. In Sec. III, we will probe the effects of the coupling parameter and other model parameters on the conductivity of the charged condensates. Finally, we will conclude in the last section of our main results.

II. GENERAL HOLOGRAPHIC SUPERCONDUCTOR MODELS IN THE NON-MINIMAL DERIVATIVE COUPLING THEORY

We will consider the action of the scalar field coupling to the Einstein's tensor $G^{\mu\nu}$ in the AdS background

$$S = \int d^4x \sqrt{-g} \left[R - \Lambda - \frac{F^2}{4} - \frac{|D_\mu \tilde{\psi} D^\mu \tilde{\psi}|}{2} - \frac{b|G^{\mu\nu} D_\mu \tilde{\psi} D_\nu \tilde{\psi}|}{2} - \frac{m^2 |\tilde{\psi}|^2}{2} - V(|\tilde{\psi}|) \right], \quad (3)$$

where $D_\mu = \partial_\mu - iA_\mu$ and b is the coupling parameter. As in [43, 44], re-scaling the charged scalar field $\tilde{\psi} = \psi e^{ip}$ with real ψ and p , and then the action (3) can be re-written in a Stückelberg form

$$S = \int d^4x \sqrt{-g} \left[R - \Lambda - \frac{F^2}{4} - \frac{(g^{\mu\nu} + bG^{\mu\nu})\partial_\mu \psi \partial_\nu \psi}{2} - \frac{m^2 \psi^2}{2} - \frac{\psi^2 (g^{\mu\nu} + bG^{\mu\nu})(\partial_\mu p - A_\mu)(\partial_\nu p - A_\nu)}{2} - V(\psi) \right], \quad (4)$$

with the gauge symmetry $A_\mu \rightarrow A_\mu + \partial_\mu \alpha$ together with $p \rightarrow p + \alpha$. Thus, the generalized Stückelberg Lagrangian action in the non-minimal derivative coupling theory can be expressed as

$$S = \int d^4x \sqrt{-g} \left[R - \Lambda - \frac{F^2}{4} - \frac{(g^{\mu\nu} + bG^{\mu\nu})\partial_\mu \psi \partial_\nu \psi}{2} - \frac{m^2 \psi^2}{2} - \frac{|\mathfrak{F}(\psi)|(g^{\mu\nu} + bG^{\mu\nu})(\partial_\mu p - A_\mu)(\partial_\nu p - A_\nu)}{2} - V(\psi) \right]. \quad (5)$$

Here $\mathfrak{F}(\psi)$ is a function of ψ , which is take as

$$\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4, \quad (6)$$

with the model parameters c_γ , γ and c_4 [43–46]. In the probe limit, the backreactions on the background metric from the matter field is negligible. Here we adopt to such a limit so that we can neglect the influence of the coupling terms contained the factor $bG^{\mu\nu}$ in the action (5) on the metric for simplicity.

One of the most simple AdS black hole is the planar Schwarzschild AdS black hole which can be described by a metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(dx^2 + dy^2), \quad (7)$$

with

$$f(r) = \frac{r^2}{L^2} - \frac{2M}{r}, \quad (8)$$

where L is the radius of AdS and M is the mass of black hole. The Hawking temperature is

$$T_H = \frac{3r_H}{4\pi L^2}, \quad (9)$$

where r_H is the event horizon of the black hole. The Einstein's tensor $G^{\mu\nu}$ in the Schwarzschild AdS black hole (7) has a form

$$G^{\mu\nu} = \frac{3}{L^2} g^{\mu\nu}. \quad (10)$$

Substituting Eqs. (7) and (10) into Eq. (5) and taking the ansatz

$$A_\mu = (\phi(r), 0, 0, 0), \quad \psi = \psi(r), \quad (11)$$

we can find that the equations of motion for the complex scalar field ψ and electrical scalar potential $\phi(r)$ can be written as

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\psi' + \frac{\phi^2}{2f^2} \frac{d\mathfrak{F}(\psi)}{d\psi} - \frac{m^2 L^4 \psi}{(L^2 + 3b)f} = 0, \quad (12)$$

and

$$\phi'' + \frac{2}{r}\phi' - (1 + 3b)\frac{\mathfrak{F}(\psi)}{f}\phi = 0, \quad (13)$$

respectively. Here a prime denotes the derivative with respect to r and we use the gauge freedom to fix $p = 0$. Obviously, the presence of the coupling factor b decreases the effective mass $m_{eff}^2 = \frac{m^2 L^2}{L^2 + 3b}$ of the scalar field and increases the current of the electric field. This means that the coupling term will change the properties of holographic superconductors in the Schwarzschild AdS black hole. In order to solve the nonlinear equations (12) and (13) numerically, we need to seek the boundary condition for ϕ and ψ near the black hole horizon $r \sim r_H$ and at the asymptotic AdS boundary $r \rightarrow \infty$. The regularity condition at the horizon gives the boundary conditions $\phi(r_H) = 0$ and $\psi = \frac{3(L^2 + 3b)r_H}{m^2 L^4} \psi'$. At the asymptotic AdS boundary $r \rightarrow \infty$, the scalar field ψ and the electric potential ϕ can be approximated as

$$\psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \quad (14)$$

and

$$\phi = \mu - \frac{\rho}{r}, \quad (15)$$

with

$$\lambda_{\pm} = \frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{m^2 L^4}{L^2 + 3b}}. \quad (16)$$

In the dual field theory, the constants μ and ρ are interpreted as the chemical potential and charge density respectively. The coefficients ψ_- and ψ_+ correspond to the vacuum expectation values of the condensate operators \mathcal{O}_{\pm} respectively. Since both of the falloffs are normalizable for ψ , one can impose the boundary condition that either ψ_- or ψ_+ vanishes, which ensures that the theory is stable in the asymptotic AdS region [55]. The choices of the boundary condition $\psi_- = 0$ or $\psi_+ = 0$ will not affect qualitatively our results. Thus,

we here set $\psi_- = 0$, $L = 1$ and $m^2 L^2 = -2$ for convenience. In doing so, we can investigate the properties of the scalar condensate $\langle \mathcal{O}_+ \rangle = \psi_+$ for fixed charge density by solving the equations of motion (12) and (13) numerically.

Let us now to study how the phase transition depends on the coupling parameter b and the model coefficients c_γ , γ and c_4 in the $\mathfrak{F}(\psi)$. In Fig. 1, we present the influence of the parameters b and c_4 on the phase transition for fixed $c_\gamma = 0$. We find that the coupling parameter b and the model parameters c_4 have obvious different effects on the critical temperature. In the case $c_4 < 1$, we find that only the second-order phase transition happens in the formation of scalar hair and the critical temperature does not depend on the coefficient c_4 . These properties of Holographic superconductors are similar to that in the case in which the scalar field is not coupling with Einstein's tensor. However, with the increase of the coupling parameter b , the critical temperature decreases, which means that the coupling between the scalar field and Einstein's tensor makes the formation of the scalar condensate more hardly. This property of the coupling parameter b has not been observed elsewhere. For the $c_4 \geq 1$, it was argued that for the case $b = 0$ the behavior of the scalar condensate means that the phase transition changes from the second order to the first order. Here we choose $c_4 = 1.2, 1.4, 1.6$ and 2.0 , and probe the effect of b on the transition point of the phase transition from the second order to the first order. Fig. 1 tells us that for the bigger value of b , the transition point appears more hardly. The critical value of b_c separating the second order and the first order phase transitions is listed in table I for selected c_4 within the range $[1.2, 2.0]$. Comparing with the discussions in [45], one can find that the influences of b on the phase transition are different from that of the Gauss-Bonnet coupling parameter α .

TABLE I: The critical value of b_c which can separate the first- and second-order behavior for different $\mathfrak{F}(\psi) = \psi^2 + c_4 \psi^4$.

c_4	1.2	1.4	1.6	2.0
b_c	0.1	0.2	0.3	0.5

We also consider the case $c_\gamma \neq 0$ for the model $\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4$. From Fig. 2, we find that the critical temperature increases with the coefficient c_γ and decreases with the power γ for fixed the coupling parameter b , which is consistent with that obtained in the case $b = 0$. For fixed the model parameters c_γ and γ , we find that the presence of b leads to the critical temperatures for two types of phase transitions more lower and makes the transition point of the phase transition from the second order to the first order appears more hardly. Moreover, we check the critical behaviors of the condensate $\langle \mathcal{O}_+ \rangle$ near the second-order phase transition point with different values of b and γ for fixed $c_\gamma = -1$ and $c_4 = 0.5$. From Fig. 3, we find the

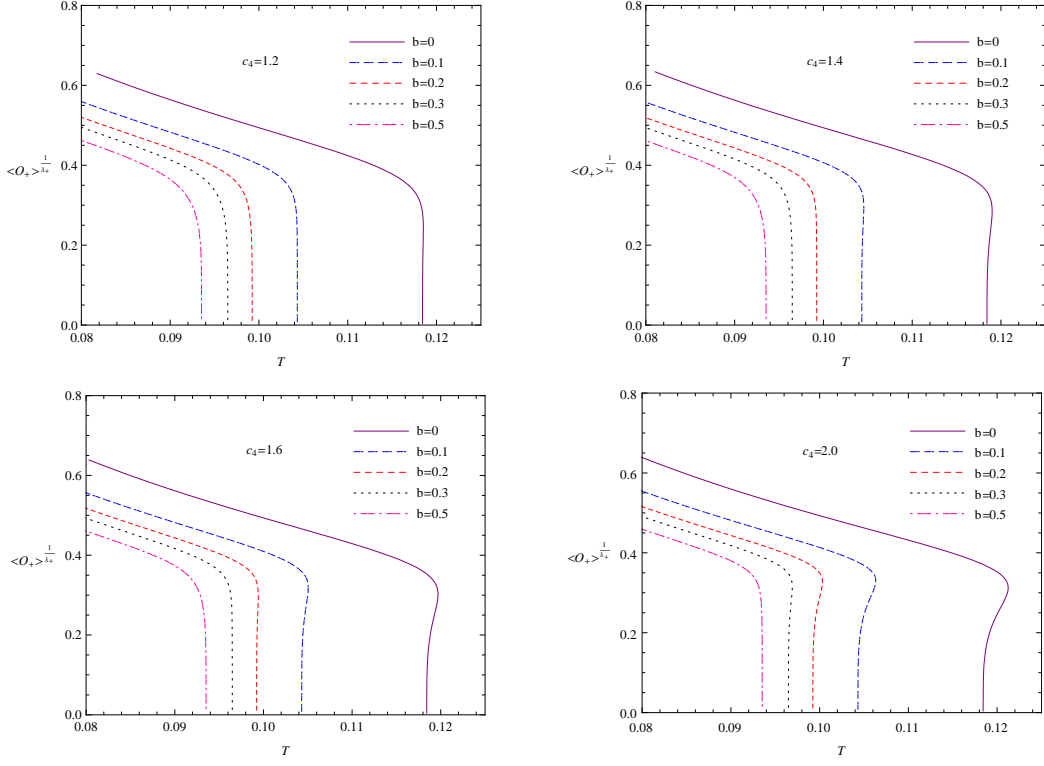


FIG. 1: The condensate $\langle \mathcal{O}_+ \rangle^{\frac{1}{\lambda_+}}$ as a function of temperature with fixed values $c_\gamma = 0$ for different values of b and c_4 , which shows that the different values of these parameters not only change the formation of the scalar hair, but also affect the separation between the first- and second-order phase transitions. The five lines in each panel from left to right correspond to decreasing b , i.e. 0.5, 0.3, 0.2, 0.1 and 0.

critical exponent β can be approximated as

$$\beta \simeq \frac{1}{\gamma - 2}. \quad (17)$$

Obviously, the critical exponent β is determined only by the model parameters in the $\mathfrak{F}(\psi)$ and is independent of the coupling parameter b , which implies that the critical exponent β does not depend on the scalar mass since the presence of the coupling parameter b changes the effective mass of the scalar field. Combining with the results obtained in [43–45, 47], we find that the dependence of the critical exponent β near the second-order phase transition point only on the model parameter γ could be a universal property in such a kind of general models for holographic superconductors.

III. THE ELECTRICAL CONDUCTIVITY

In this section we will investigate the electrical conductivity when the scalar field coupling to Einstein's tensor and see what effects of the coupling parameter b and the model parameters (c_γ , γ and c_4) on the

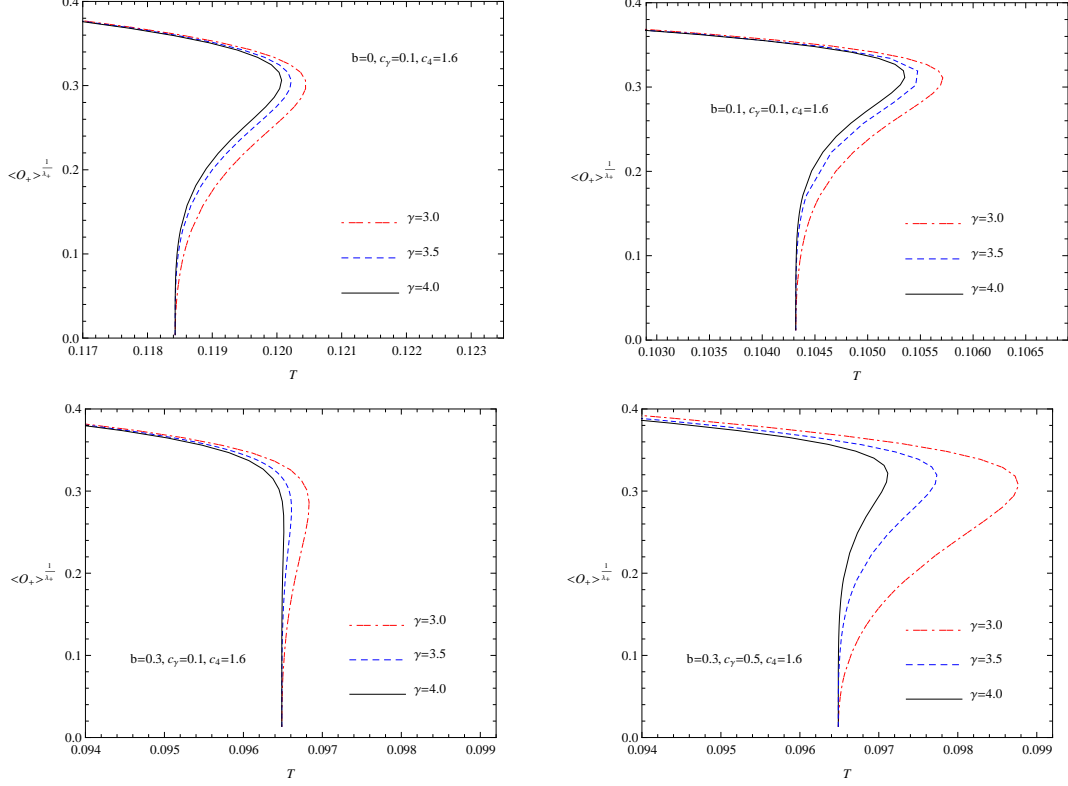


FIG. 2: The condensate $\langle \mathcal{O}_+ \rangle^{\frac{1}{\lambda_+}}$ as a function of temperature with fixed values c_γ for different values of b and γ , which shows that the type of phase transition depends on the coupling parameter b , model parameters c_γ and γ . The three lines in each panel from left to right correspond to decreasing γ , i.e. 4, 3.5 and 3.

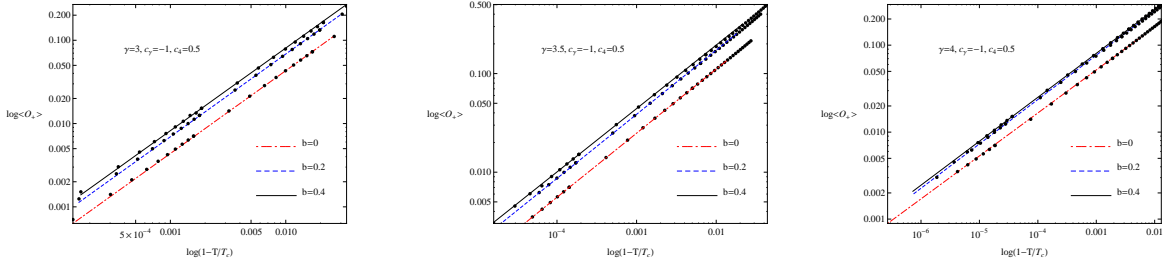


FIG. 3: The condensate $\langle \mathcal{O}_+ \rangle$ vs $1 - T/T_c$ in logarithmic scale with different values of b and γ , which shows that the slope is independent of b but sensitive to γ .

electrical conductivity in the planar Schwarzschild AdS black hole spacetime.

Following the standard procedure in [4–8], we suppose that the perturbation of the vector potential is translational symmetric and has a time dependence as $\delta A_x = A_x(r)e^{-i\omega t}$. In the probe approximation, the effect of the perturbation of metric can be ignored. Thus, equation of motion for δA_x obeys to

$$A_x'' + \frac{f'}{f} A_x' + \left[\frac{\omega^2}{f^2} - (1 + 3b) \frac{\mathfrak{F}(\psi)}{f} \right] A_x = 0. \quad (18)$$

As b tends to zero, the above equation can be reduced to that in the case in which the scalar field is not coupling with Einstein's tensor. Since there exists only the ingoing wave at the black hole horizon, the boundary condition on A_x near $r \sim r_H$ is $A_x = f^{-\frac{i\omega}{3r_H}}$. In the asymptotic AdS region, one can find easily that A_x behaves like

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r}. \quad (19)$$

According to the AdS/CFT, we can express the conductivity as

$$\sigma(\omega) = \frac{\langle J_x \rangle}{E_x} = -\frac{i\langle J_x \rangle}{\omega A_x} = -i \frac{A_x^{(1)}}{\omega A_x^{(0)}}. \quad (20)$$

Solving the motion equation (18) numerically, we can obtain the conductivity for the general forms of function

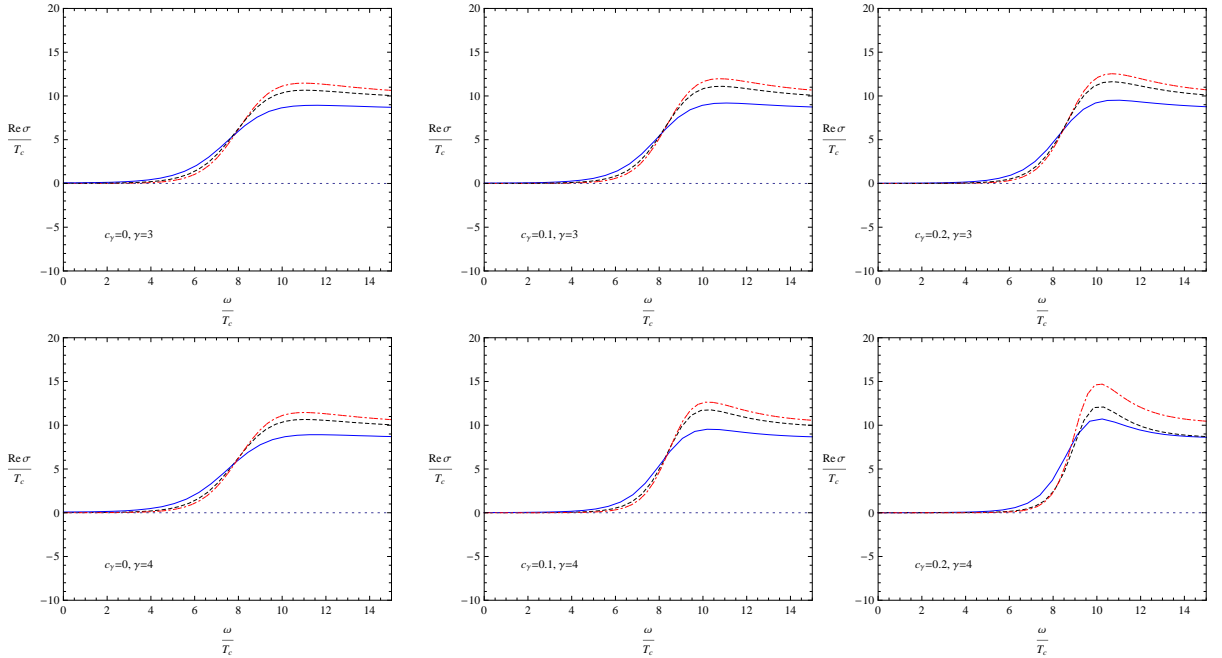


FIG. 4: The real part of of the conductivity with fixed values of b for different models with $\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma$. The solid, dashed and dash-dotted lines are corresponding to the cases with $b = 0, 0.1$ and 0.2 , respectively.

$\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4$. Here we set $c_4 = 0$ and $m^2 L^2 = -2$ for clarity.

In Figs. 4 and 5, we plot the frequency dependent conductivity for operator $\langle \mathcal{O}_+ \rangle$ with different values of b , c_γ and γ . Fixing the coupling parameter b , the gap frequency ω_g increase with c_γ for fixed γ , grows with γ for fixed c_γ . These behaviors of the gap frequency ω_g are also observed in the case $b = 0$. For fixed model parameter c_γ and γ , we find that the gap frequency ω_g becomes bigger for the larger b . It implies that the presence of the coupling parameter b enhances the strength of the strong coupling in holographic

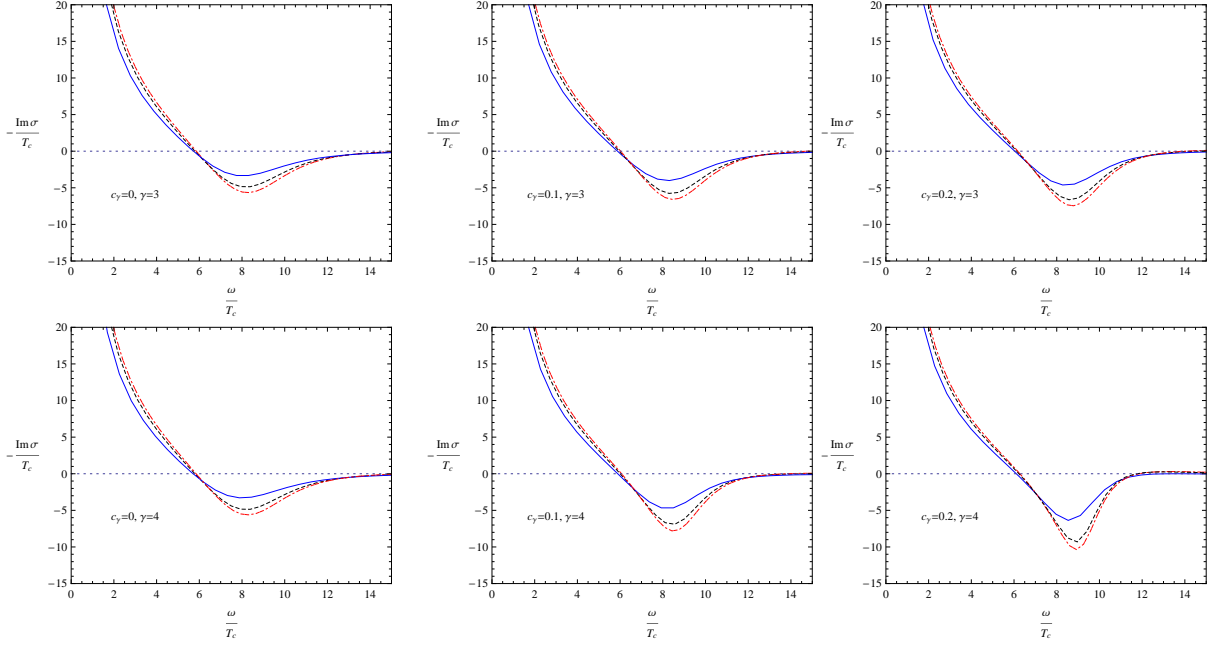


FIG. 5: The imaginary part of the conductivity with fixed values of b for different models with $\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma$. The solid, dashed and dash-dotted lines are corresponding to the cases with $b = 0, 0.1$ and 0.2 , respectively.

superconductors. Our results show that not only the form of the scalar field $\mathfrak{F}(\psi)$, but also the coupling between the scalar field and Einstein's tensor of the background will affect the so-called universal relation $\omega_g/T_c \simeq 8$.

IV. SUMMARY

In this paper we studied a general class of the holographic superconductor models via the Stückelberg mechanism in the non-minimal derivative coupling theory in which the charged scalar field is kinetically coupling with Einstein's tensor. In the probe limit, we found that the coupling parameter b decreases the critical temperature and makes the formation of the scalar condensate more hardly. Our results also show that both the coupling parameter and the parameters which define $\mathfrak{F}(\psi)$ can separate the first and second order phase transitions. With the increase of b , the transition point of the phase transition from the second order to the first order also appears hardly in this generalized system. This means that the influences of b on the phase transition are different from that of the Gauss-Bonnet coupling parameter α . For the second order phase transition, we observed that the deviation of the critical exponents from that of the mean field result is independent of the coupling parameter b and is determined only by the model parameters in the $\mathfrak{F}(\psi)$.

We also calculated the electric conductivity numerically and found that the gap frequency ω_g depends not

only on the form of the scalar field $\mathfrak{F}(\psi)$, but also on the coupling between the scalar field and Einstein's tensor of the background. With increase of the coupling parameter b , the ratio of the gap frequency in conductivity ω_g to the critical temperature T_c increases, which implies that the coupling between the scalar field and Einstein's tensor enhances the strength of the strong coupling in holographic superconductors.

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